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Book Review: *Fractals: Form, Chance, and Dimension*

Fractals: Form, Chance, and Dimension. By Benoit B. Mandelbrot. Freeman, San Francisco, 1977, $14.95.

This is an essay, in the old-fashioned meaning of the word, on the prevalence of "fractal" shapes in Nature. The adjective fractal, coined by the author, is by preference not defined too precisely. It is intended to convey the idea of a complicated, possibly self-similar figure whose ordinary topological dimension d is smaller than its effective dimension D from a metric point of view. The latter may be given a precise meaning as follows. Let the figure be situated in, e.g., three-dimensional space and let the number e be chosen at pleasure. Then there is a critical number $D =$ *the fractal dimension of the figure*, lying between 0 and 3, inclusive, such that (a) if $e > D$, the figure can be covered by balls of radii $r_1, r_2, r_3, \ldots$ so as to make $M = r_1^e + r_2^e + r_3^e + \cdots$ as small as you like; (b) if $e < D$, then $M = \infty$ for every such covering no matter how tight. D is never less than the topological dimension d of the figure: $D \geqslant 1$ for curves, $D \geqslant 2$ for surfaces, etc.

The simplest examples of fractal figures are the standard Cantor set in $0 \leqslant x \leqslant 1$ with $d = 0 < D = \lg 3/\lg 2$, and the locus of the standard Brownian motion in two or three dimensions with $d = 1 < D = 2$. The examples cited from Nature come from an extraordinary variety of sources: (a) the coastline, e.g., of Great Britain, (b) the intermittent nature of turbulence likened to a Cantor set, (c) the boundary of the region of turbulent flow, e.g., the surface of a cloud, (d) the locus of stellar matter in space, (e) the curdling of milk and the interface between curds and whey so produced, (f) the shape of a polymer compared to a self-avoiding random walk, (g) the pattern of rivers and drainage basins. Clearly, this is a very rich array of natural geometrical figures, which Euclidean geometry and its standard figures give us no ready way of describing. It is this gap that fractal figures and their morphology are supposed to fill. The essay is full of pictures of such things produced by computer simulation. I should have liked to see more figures from Nature as well and a detailed comparison of the former with the latter. This is not provided here, but many examples may be found in the author's papers. The discussion is continually stimulating and provocative and presented in

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an entertaining style. I think the author has proved at least part of his case: *that fractal figures abound in Nature*, and I recommend this essay to anyone who deals with such matters, if only as a spur to substantiate the claim more fully on actual physical phenomena.

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Book Review: *Classical Kinetic Theory of Fluids*

Classical Kinetic Theory of Fluids. By P. Résibois and M. De Leener. Wiley–Interscience, New York, 1977, $31.95.

In spite of a great deal of active and productive research during the past two decades, the number of books on the principles of nonequilibrium statistical mechanics and the description of dynamical properties of macroscopic systems is very small. There are specialized monographs, review articles, and lecture note series, but most often these are inadequate to serve as a text for a course or as general reading for a graduate student starting research in the area. Therefore, the book by Résibois and De Leener is a timely and valuable contribution to partially fill this relative void in the physics literature. Professors Résibois and De Leener have made important contributions to the development of this subject and consequently there are more substantial reasons why this book will be useful to both students and research scientists alike.

The scope of the book is quite large, although it is limited primarily to simple atomic fluids in the classical regime. This is not a limitation in principle but is a simplification that often allows a somewhat greater exposure of the detailed relationship between the microscopic laws and macroscopic dynamics. To many, the terminology "kinetic theory" means the Boltzmann equation and its corresponding limitations (e.g., low density, space and time scales large compared to corresponding atomic dimensions). Recent developments, however, have emphasized generalizing the concepts of the Boltzmann kinetic theory to apply to dense gases and liquids, with substantial success. Approximately two-thirds of the book is devoted to a description of such work, with attention to developments of a conceptual nature as well as improved quantitative analysis. The first third of the book contains a review of stochastic processes as an application of the notions of statistics and probability, with applications limited mainly to Brownian motion. The presentation is clear and self-contained but limited by brevity. Some mention of the existence of a statistical mechanical framework to study the basis for the stochastic models (for example, as described in the book by DeGroot and Mazure) would have provided better continuity with the latter part of the

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book. Nevertheless, the authors are to be credited for choosing to include stochastic processes in a book on kinetic theory. Also contained in the first third of the book is a discussion of the Boltzmann equation, its linearization, and the Chapman–Enskog solution. This section is very well done and constitutes a good overview of the more complete books of Chapman and Cowling and of Ferziger and Kaper. Many of the results, such as hydrodynamic modes, eigenvalues, and eigenfunctions, are obtained in this section for later use in the dense gas kinetic theory.

The middle third of the book is devoted to an exposition of generalized kinetic equations for the time dependence of the singlet distribution function. This section is less satisfactory than one might have hoped, due mainly to the authors' attempt to describe the general case of nonlinear kinetic theory. To make this manageable, only the spatially homogeneous case is considered. A corresponding formal master equation is derived, and the desired kinetic equation obtained via an assumption (i.e., that the N-particle, spatially averaged distribution function factors for all times into a product of single-particle distribution functions). While it is true that the assumptions do not refer to low density as in the Boltzmann case, the reader is left with the feeling that great damage may have been done to the relationship between the resulting generalized kinetic equation and the underlying laws. The justification for the basic assumption (VIII.41 or VIII.57) includes a comment about its proof for a model master equation by Kac; however, the proof is neither given nor discussed. Since the generalized kinetic equation is a fundamental topic of the book and its application is discussed extensively, the three pages devoted to this one key assumption hardly seem enough. Limitation to conditions for linear kinetic equations might have provided a more convincing case for generalized kinetic equations, since in that case the factorization assumption is not required (as indicated in the section on time correlation functions). The sections on application of the kinetic equation are brief but clear, making contact with the previously discussed Fokker–Planck and Boltzmann equations. This discussion on applications concludes with a chapter on hard-sphere dynamics and the formal density expansion of transport coefficients. Both the physics and mathematics responsible for the failure of the expansion, and the ring resummation, are well described.

The final section on time correlation functions provides both motivation for their introduction and their connection with experiment. The chapter on calculation of time correlation functions starts out with a description in terms of the generalized kinetic equation, but this is quickly abandoned with the unqualified comment, "The formal approach ... is of no help for dense fluids ...." It would have been nice if the subsequent phenomenological methods could have been related to or generated from models for the collision operator. For example, the various exact memory functions are introduced, but their relationship to kinetic theory is not given. On the positive side, the authors present a coherent sampling of a wide variety of topics referring to the reponse and fluctuations of liquids, including a short section on computer experiments. The final section returns to the ring kinetic theory for a description of the long-time tails of correlation functions, and is again well done.

On the whole, as noted at the outset, this book is worthwhile reading for physicists concerned with these matters and will certainly serve well as an introduction for graduate students to a fascinating subject; the authors are experts in this field and one should pay attention to the experts. The reservations mentioned above are given primarily to indicate that the book is not as complete on any given topic as one might hope, perhaps due to the scope considered. There is one flaw, however, that will be apparent to both student and practitioner alike, and that is the disregard for adequate references. The authors state that the literature is too voluminous to cite, and restrict themselves to selected books, monographs, and review papers. On the contrary, citations are most useful when the literature is extensive. A student or newcomer must now do his own "spade work" to find original papers, opposing points of view, or up-to-date developments. A considerable value would have been added had the authors included somewhat more than the three pages of references for the entire book.

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